

<chapter heading: commentary>

7.7-8 Ἀντιλήψεις τῶν μαθημάτων ὡς οὐδενὸς ἀξίων ὄντων, καὶ ἀντιλογίαι: The title announces two main topics: objections to theoretical mathematics (corresponding to 79.1-81.4); and then responses to those objections (corresponding to 81.5-84.20). This division should be seen as corresponding to a change of speakers: it is the speaker ‘Isocrates’ who provides the objections to abstract mathematics, and ‘Aristotle’ who provides the replies to these objections (for the identification of characters, see <xx>).

The objections are transmitted in chapter 26 of the *DCM* only, whereas the replies are transmitted also in *Protr.* VI. The two versions of the replies of ‘Aristotle’ differ in that at 81.5-83.2 the *DCM* is somewhat condensed relative to the fuller version that occupies *Protrepticus* VI from 37.26 to the end of the chapter, and the *DCM* provides a more extensive quotation from Aristotle’s work at 83.2-84.20, a passage not cited at all in *Protr.* VI. Thus from the evidence point of view, the chapter consists of three sections, the middle one overlapping with *Protrepticus* VI, and in the outer two the *DCM* chapter being the sole witness.

The history of attributions from the chapter shows a piecemeal approach; no one seems to have considered the chapter as a whole and what its structure reveals about how Iamblichus used his source, nor were the overlaps with *Protrepticus* VI studied closely and understood. Rose in his 1886 edition of the fragments included two parts of *DCM* XXVI: 79.1-81.7 (= Fragment 52; 5b Walzer; 5 Ross;) and 83.6-13 (= Fragment 53; 8 Walzer/Ross). Neither Rose nor Walzer/Ross included the rest of *DCM* XXVI (81.7-83.5 and 83.23-84.20), and so not *DCM* XXVI 82.17-83.2, even though that range of *DCM* XXVI is an exact overlap—word for word identical—with *Protrepticus* VI 40.12-41.2. This is not much of a loss, since *DCM* XXVI preserves a subset of the excerption of *Protrepticus* VI, and so there seemed to be nothing gained by adding in this material. (In fact, one crucial piece of information about dialogue may be preserved in the *DCM* but not *Protr.*: see note on 82.17. On the other hand, the last section (83.23-84.20) should not continue to be ignored, as it appears to preserve a complete argument not found in *Protrepticus* VI or anywhere else. Düring made a serious mistake in rejecting *DCM* 26 *in toto* as evidence for Aristotle’s *Protrepticus*.

Proclus mentions ideas from throughout the chapter in his *Commentary on Euclid I* 1, chapter 9, and he attributes two specific arguments to ‘Aristotle’. As Francesco Romano commented, with respect to the whole stretch of 79.1-83.22 (which stretch we attribute to Aristotle, except where we detect Iamblichean intervention): “cf. Aristotle, *Protr.* B52 Düring, partialmente. Cf. anche Proclo, *In. Eucl.* 25.15-29.13, che si riferisce quasi certamente al Prorettico di Aristotele, come suggerisce il πείθειν di 26.13” which is also pointed out in notes to the translation of Proclus’ *Euclid* commentary in Morrow, p. 22n49. We follow the attributions of Rose, Walzer, Ross, Morrow and Romano in recognizing Aristotle’s *Protrepticus* as the source of this chapter, but our structural approach to the whole chapter allows us to discriminate with more precision how the cover text relates to the source text.

7.7 Ἀντιλήψεις: Cf. *DCM* XXXV 98.17.

<79.1-15: commentary>

79.1-5: attribution and voice: 79.1-5 appears to be an opening summary of the chapter by Iamblichus, although none of the terminology is anachronistic and some of it may have been borrowed from his source.

79.1-3 οἱ μὲν παλαιοὶ οἱ δὲ νέοι, οἵτινες τὴν ἐναντίαν δόξαν περὶ τῶν μαθημάτων ἐξενηνόχασι: Iamblichus here announces a change in voice, calling to mind certain people that have detracted from mathematics. He seems to signal that he is now quoting from a different voice, but his source remains the same work, since it clearly connects with both the earlier and later material in the chapter (and with the rest of the *Protrepticus* material).

Proclus discusses these two kinds of detractors of mathematics, but does not differentiate them on the basis of their antiquity; in lieu of this, he distinguishes between those who deny that the beautiful exists in mathematics, and those who deny the usefulness of mathematics (*In Euc. I* 25.12-23; see text, translation and commentary on pp. NN). In *DCM XXVI* we hear nothing further in terms of Iamblichus' distinction (the arguments against mathematics are not described as being either ancient or contemporary); and we hear next to nothing of Proclus' first group, those who deny the existence of beauty in mathematics. Iamblichus' excerpts focus almost exclusively on the arguments criticizing the usefulness of mathematics. Thus it may be that Iamblichus is imposing his own temporal distinction on the two groups. On the other hand, the argument against the existence of beauty in mathematics may have been attributed to an earlier philosopher (such as Aristippus), while the argument against the utility of theoretical mathematics could be attributed to a relatively more recent (indeed, contemporary), such as Isocrates.

Aristotle in *Metaph.* III 2 mentions Aristippus as an example of a detractor of mathematics because of its disconnection from goods: “in mathematics nothing is proved by means of this kind of cause, nor is there any demonstration of this kind—‘because it is better, or worse’; indeed, no one even mentions anything of the kind. And so for this reason some of the Sophists, e.g. Aristippus, ridiculed mathematics; for in the arts, even in handicrafts, e.g. in carpentry and cobbling, the reason is always given ‘because it is better, or worse’, but the mathematical sciences take no account of goods and evils” (996a29-b1, ROT; cf. DL II.79-80; Aristippus is also mentioned in P.Oxy. 3659, which may have been a part of the *Protrepticus*. For carpentry, cf. *Protr.* X.54.22-55.7. For cobblers, cf. Stob. *Eclogues* IV.32.21). This line of argument is rejected in *Metaph.* XIII 3, where Aristotle argues that the good or beautiful exists in mathematics, in fact here most of all, since mathematical objects above all manifest order, symmetry, and definiteness (1078a33-b6). Proclus preserves a similar argument in a part of his commentary (*In Euclid I*, prologue I, chapter 9.26-27), but he describes his source as the principles by which Aristotle attempts to persuade us” (see our text, translation, and commentary on pp. NN), which Morrow, Romano, and we take to indicate that Aristotle's *Protrepticus* is the source rather than the *Metaphysics*.

As for the argument criticizing the uselessness of mathematics, Diogenes of Sinope reportedly “held that one should avoid music, geometry, astronomy and that kind of thing as useless and unnecessary (ἀχρήστων καὶ οὐκ ἀναγκαίων)” (DL VI.73; cf. VI.28). Aristotle in the *Protrepticus* is most likely to be responding to the diatribe against theoretical preoccupations in education in *Antidosis* 261-269 where Isocrates calls into question “those who are occupied with astronomy and geometry and studies of that sort” (*Antid.* 261). Isocrates relates a complaint about the “uselessness” of these kinds of study: “most men see in such studies nothing but empty talk and hair-splitting; for none of these disciplines has any useful application either to private or to public affairs; nay, they are not even remembered for any length of time after they are learned because they do not attend to us through life nor do they lend aid in what we do, but are wholly divorced from our necessities” (*Antid.* 262; see our essay

‘*Antidosis* and *Protreptius*’ on pp. NN for a fuller treatment of this passage). Although Isocrates’ own remarks are nuanced and his target is the educational scheme of the Academy and Plato’s defense of a curriculum focused on such mathematical studies (of the kind we find at *Rep.* VII), his remarks indicate that this is a more generally held view about mathematical and theoretical studies, in fact one that “most men” share.

Even after Aristotle, mathematical astronomy was also criticized for its uselessness by Epicurus; see *Ep. Pyth.* D.L. X.86-87, 93-94, 97-98, 114, 116, and passim. In fact, an anonymous scholar who was reading this passage in manuscript F of Iamblichus’ *DCM* left a marginal remark warning against preoccupation with mathematics: “Epicurus said, ‘you should make off with the speedboats, and flee from mathematics’,” adapting or misremembering a citation from a *Letter to Pythocles* found in Diogenes Laertius X.6 (fr. 163 Usener): “Make off with the speedboat, my dear fellow, and flee from all culture!”

79.5 ἐπιχειροῦσιν: see below ἐπεχείρησαν φιλοσοφεῖν at 83.12. The verb is an Academic term of art, meaning “attempt to prove” or “argue dialectically” (e.g., Pl. *Tht.* 205a, Ar. *APr.* 66a34, *Top.* 101a30, 128b26, see LSJ s.v. III). In the *Topics*, Aristotle actually designates it a technical term for a dialectical as opposed to an apodeictic proof (*Top.* 162a16 and this is followed in the *Rhetoric*).

79.5-15: attribution and voice: Here, Aristotle apparently wrote lines for ‘Isocrates’ criticizing the purely theoretical sciences from the perspective of the practical sciences. In our view, this follows on a part of a speech excerpted by Iamblichus at *Protr.* VI. 37.11-22, in which the distinction is made between practical and productive sciences, with practical sciences judged to be “more commanding” and superior than the productive sciences. The speech from *DCM* 79.5-24 then sets up the argument that the practical sciences are also more valuable than the purely theoretical sciences, and continues in the same voice from 79.5-81.4. On Isocrates as a critic of the uselessness of theoretical philosophy and mathematics, see note above on 79.1-3.

79.5-8 εἰ ἀχρεῖον αὐτῶν τὸ τέλος, δι’ ὅπερ αὐτὰ μανθάνειν φασὶ δεῖν οἱ φιλόσοφοι, πολὺ πρότερον ἀνάγκη μάταιον εἶναι τὴν περὶ ταῦτα σπουδῆν: The framework for this discussion seems to be very similar to one of the protreptic sections of Plato’s *Euthydemus*, in which the agreement that “it is necessary to do philosophy” (288d) is examined, and productive and practical kinds of knowledge are distinguished (289a), and mathematics is instanced as the former because mathematicians do not know how to make use of any of their mathematical discoveries (290c).

79.9-10 μάλιστα ἡκριβωκένα: See below ἡ περὶ τὴν ἀλήθειαν ἀκριβολογία at 83.7 and κάλλει καὶ ἀκριβεία at 83.24-25 and the note on ἀκριβέστερος at *Protr.* VI 39.17 (and cf. VI 40.19 and *DCM* XXVI 79.10, 83.7, 83.24-25, *DCM* XXVII 86.8).

79.10-14 οἱ μὲν ... οἱ δὲ: The people that identify the end of philosophy with intelligence about nature and reality are here exemplified by Anaxagoras and Parmenides. But who would Aristotle consider representative of the other view mentioned here, that the end of philosophy is knowledge of the just and the unjust? In the *Parts of Animals* I, a book with several other clear links to the *Protrepticus*, Aristotle draws a distinction between the kind of pursuit exemplified by Democritus on the one hand and, on the other hand, the one of Socrates, who turned exclusively to practical and moral issues: “The cause of our predecessors not proceeding in this manner is that they did not have the cause for the sake of which, i.e. the definition. Democritus was the first to approach it, but he did not require it in his physical theory, instead he was brought to it by the fact of the matter itself. In the time of Socrates, a nearer approach was made, but the search for the things concerned with nature was given up, and the philosophers turned their attention towards the usefulness of virtue and the political” (*PA* I

1.642a24-31; cf. *Metaph.* XIII 4.1078b17-23). Notice that in this passage, Democritus' physical (and possibly mathematical) investigations are said to lack reference to the "cause for the sake of which", that is, any reference to "better or worse"; while Socrates, who held the investigations of the good and the virtues to be paramount, turned away from natural science and in particular that of none other than Anaxagoras (as Plato depicts him doing in *Phd.* 98b7-99b4).

79.12-13: τὴν περὶ φύσεώς τε καὶ τῆς τοιαύτης ἀληθείας φρόνησιν: Cf. *Protr.* VI 38.1-2: περὶ φύσεώς τε καὶ τῆς ἄλλης ἀληθείας ἐπιστήμας. Notice the two similar but different phrases in the mouth of the two different characters: at *Protr.* VI 38.1-2, 'Aristotle' speaks of the "sciences concerning nature and other sorts of truth", whereas 'Isocrates' prefers to refer to the "intelligence concerning nature as well as truth of that sort" (*DCM* 26 79.12-13). See note at ??? on the repeated substitution of Aristotle's preferred term "wisdom" in place of Isocrates' preferred term "intelligence".

79.14: οἷ τε περὶ Ἀναξαγόραν καὶ Παρμενίδην: Burkert cites this passage as evidence (originating from the *Protrepticus*) that Aristotle did not consider Pythagoras an important individual in the history of theoretical philosophy. In a note, he adds: "the very fact that Thales and Pythagoras are not named in this passage is an indication of the Aristotelian origin of the wording" (*Lore and Science*, 216n31).

Burkert also notes that Isocrates credits Pythagoras with being "the first to introduce all the other philosophy to the Greeks" (*Busiris* 28) but specifies this contribution as concerned with "sacrifices and temple rites", not astronomy, calculation and geometry (referred to in *Busiris* 23). It should be pointed out, however, that it was appropriate for Aristotle in the voice of Isocrates to specifically name Anaxagoras and Parmenides (and their followers) as devotees of the philosophy of nature and "that sort of truth", since these were among the figures whom Isocrates had ridiculed in his *Antidosis* (268-9), where Parmenides is mentioned by name as a monist, and those who believe in an infinite plurality of first principles are also mentioned, although Anaxagoras is not explicitly named.

<79.15-80.1: commentary>

79.15-24: attribution and voice: Aristotle's formulation of the utilitarian principle using the classic examples of health, wealth, and in general success, was presumably put into the voice of a character, like Isocrates, who held just such a principle.

79.19-24: Cf. Proclus in *Euc.* I, Prologue I, chapter 9: "**we are not wealthy by knowing about wealth** but by using it, nor are we successful by knowing about success but by living successfully; hence with respect to the human way of life as well our actions, we will agree, etc." (25.23f.). Proclus mentions wealth and *eudaimonia*, but dropped the health example from his paraphrase.

79.24-80.1: attribution and voice: It is not clear to what extent the conclusion at 79.24-80.1 has suffered from compression or paraphrase, and thus we keep it in straight text.

<80.1-13: commentary>

80.1-5 attribution: a typical bridge passage of Iamblichus containing signposts, repeating the conclusion of the preceding arguments that mathematics is not a productive science, and

announcing the forthcoming arguments that mathematics is not a practical science. This conclusion serves as the minor premise of the overall argument of the speech attacking theoretical philosophy.

80.5-13 attribution and voice: This is a continuation of the speech attacking mathematics (in the voice of “Isocrates”). That Aristotle is the author is clear from the specificity of the examples used to make the point, namely geometry (80.5-13), music (80.13-23), and astronomy (80.23-81.4), on which see the next note. That they are appropriate to put in the voice of Isocrates is suggested by Isocrates’ reference to ἀστρολογίᾳ καὶ λογισμοῖς καὶ γεωμετρίᾳ in *Busiris* 23. Burkert (*Lore and Science*, 422) argues that Isocrates does not mean to connect Pythagoras with these studies when he mentions Pythagoras “being the first to bring all philosophy to the Greeks” (*Busiris* 28), but this is contradicted by Zhmud, *Early Pythagoreans*, 49-50. That dispute does not affect whether Aristotle could have put into the voice of the character Isocrates a discussion of Pythagoreanism making reference to those sciences, which were developing rapidly in his own day (see below 83.13-23 and note *ad loc.*).

80.5-6: τὰς ὁμοίας ἐπιστήμας αὐτῇ καὶ τὰς ὑποκειμένας δόξας
The examples in Aristotle’s *Protrepticus* are also precisely those found in his discussions of “the more physical of the mathematical sciences” in *Physics* II 2.194a7-12 (examples: geometry, harmonics, and astronomy). In the *Posterior Analytics*, these sciences are described as being in a hierarchical relationship in which the empirical science is “under” (*hypo*) the mathematical science; *Posterior Analytics* I 7.75b12-20 (examples: optics-geometry; and harmonics-arithmetic) and I 13.78b32-79a7 (examples: optics-geometry; mechanics-stereometry; harmonics-arithmetic; navigation-astronomy).

Philosophers of science still refer to “subalternate” or “subordinate” sciences. Aristotle posits these when there is a division of labor between an empirical science that documents the phenomena or facts (such as the risings and settings of certain stars, or the relative pitch of sounds), and a superordinate mathematical science (e.g., stereometry and arithmetic) which explains these phenomena by referring to causes and, ultimately, mathematical principles. Thus optical, musical, and mechanical phenomena will ultimately be explained through geometrical, arithmetical, and stereometric principles. Elsewhere in the *Posterior Analytics*, Aristotle seems to suggest a threefold scheme whereby the lowest level science involves the empirical identification of facts (e.g., about the rainbow) while an intermediary mathematical-physical science provides theorems (e.g., about the nature of optical rays and their reflection in mirrors), but this intermediate physical-mathematical science is in turn subordinate to a purely mathematical sciences, geometry. In this way, Aristotle envisions explaining empirical facts by means of mathematical principles. For a discussion of this kind of explanation, see McKirahan, “Subordinate Sciences”; Hankinson, ‘Kind-crossing’; and Johnson, ‘Explanation of the Halo’.

Aristotle in his extant works investigated the methods and structures of these very sciences, and conceived of the practical or productive sciences (like land-surveying, musical performance, navigational astronomy, and mechanics) as empirical or physical sciences subordinate to purely mathematical and theoretical sciences like geometry, arithmetic, and stereometry. In Aristotle’s extant works these are not merely examples of sciences with a certain structure, but models of scientific knowledge, whereby it can be seen how knowledge of empirical reality can be demonstrated by relating the data to mathematical principles. But in the *Protrepticus* Aristotle considered arguments about the superiority of the more mundane empirical sciences themselves on the basis of their utility, arguments apparently put into the

mouth of a critic of purely theoretical philosophy like Isocrates. Thus we consider all of the following examples of the superiority of practical sciences to their mathematical counterparts to be in the voice of Isocrates.

80.6 τὰς ὑποκειμένους δόξας: in the Isocratean philosophy of science, the units of a science are the bits of dogma that comprise it; Aristotle expresses this conception by using the phrase “the opinions that underlie” the empirical sciences that are similar to the mathematical ones.

<80.13-81.4: commentary>

80.13-81.4 attribution and voice: The speech attacking mathematics continues (in the voice of “Isocrates”), using the Aristotelian examples of mathematical sciences: music and astronomy (see above note on 80.5-6). The examples of sciences with an empirical and a mathematical aspect discussed in the surviving parts of Aristotle’s *Protrepticus* include geometry (80.5-13), music (80.15-23) and astronomy (80.23-81.4).

80.13-14 καὶ μουσικὴν καὶ τὰς ἄλλας ἐπιστήμας: see also below μουσικὴν καὶ τὰς τοιαύτας τέχνας at 83.11, where music is mentioned along with the other arts that developed for the sake of pleasure once early humans had taken care of necessities like food. Isocrates has no hesitation calling music a science, but this is not Aristotle’s view; for him, it was a technical skill that was expressed in the production of music, not a knowledge of any field. On the ideas in these passages, see Richter, ‘Musik im Aristotelischen *Protreptikos*’.

80.14-15 ὅσαις διήρηται τό τε τῆς γνώσεως καὶ τὸ τῆς ἐμπειρίας χωρίς: On the distinction (and separation and hierarchical ordering) between “cognitive” and “empirical” sciences, see above note at 80.5-6. The *Protrepticus* thus seems to contain one of the earliest attempts to precisely differentiate the empirical from other sciences (as opposed to distinguishing between science *tout court* and empirical apprehension, as in Plato).

80.15-16 οἱ μὲν γὰρ τὰς ἀποδείξεις καὶ τοὺς συλλογισμοὺς διωρισμένοι περὶ συμφωνίας: Aristotle discusses a science of harmonics in a subordinate relationship to arithmetic in the first book of the *Posterior Analytics* (I 7.75a38ff, 9.75b40ff, 13.78b32ff, 27.87a31ff).

80.23 ὅλα καὶ παντὶ: the same expression as Pl. *Rep.* VII 527c7.

80.24 κατὰ τὴν ἀστρολογίαν: See also τῆς ἀστρολογικῆς ἐπιστήμης at *DCM* XXIII 72.18-19. Iamblichus’ usual term is ἀστρολογικῆς (*DCM* 19.1, 47.15, 86.16). The use of the older term only here and at *DCM* XXIII is probably not due to mere variation in terminology on the part of Iamblichus, but strongly suggests that the source text used the older term, exactly as Aristotle would have. This linkage between *DCM* XXIII and XXVI supports the *en bloc* attribution of *DCM* XXIII-XXVI to Aristotle. Compare *DCM* XXVII: ἀστρονομία (85.16). See Simplicius, *in Phys.* 293.11-16 (Diels), for a discussion of the terminology (and Bechtle, *Iamblichus: Aspekte*, 47; Mansfeld, *Proleg. Math.*, 1n3).

80.24-25 ἡλίου καὶ σελήνης καὶ πέρι τῶν ἄλλων ἄστρον: cf. *Protr.* IX: τὰ περὶ τὸν οὐρανὸν καὶ περὶ αὐτὸν ἄστρα τε καὶ σελήνην καὶ ἥλιον (51.13-15). These words are attributed to Anaxagoras, mentioned just a few lines above at 79.14, by whoever is speaking in *Protrepticus* IX 51.11-15. Anaxagoras is there said to hold that the purpose of human existence is the observation of the heavenly bodies and the things concerned with the stars including the moon and sun. This suggests that he is also type of philosopher under consideration here in *DCM*, notwithstanding his reputation as a philosopher effective and

influential in the political sphere as well. The order “sun and moon and other stars” found in the *DCM* (but not in the *Protr.*) is the same order as in the definition of astronomy found in Ptolemy, *Apotelesmatica* I 1.2.19-21. See Bechtle’s note *ad loc.*

80.28 ναυτικὰς καλουμένας ἐπιστήμας: Aristotle distinguishes between mathematical astronomy and an empirically oriented science of “nautical astronomy” in *Posterior Analytics* I 13 (78b39-79a1; cf. *APr.* 46a19-21) For “mathematical astronomy” see *Phys.* II 2.194a8-10 (cf. *Metaph.* I 8.989b29-34, III 2.997b15-23; XII 8.1073b3-8, XIII 2.1077a1-9). Plato makes a similar distinction at *Rep.* VII 529c4-531c8.

81.2-4 εἰ δὲ τῶν πράξεων τῶν ὀρθῶν ἀπολείπονται, τῶν μεγίστων ἀγαθῶν ἀπολείπεται ἢ φιλομάθεια: In *NE* X 3, Aristotle also compares the love of music to the love of learning (ἢ φιλομάθεια): “One might think that all men desire pleasure because they all aim at life; life is an activity, and each man is active about those things and with those faculties that he loves most; e.g. the musician is active with his hearing with respect to tunes, the lover of learning with his intellect with respect to theoretical things (ὁ φιλομαθῆς τῆ διανοίᾳ περὶ τὰ θεωρήματα)” (1175a10-15, ROT adapted; notice also the relevance of this passage for the main argument of *Protr.* XI). The Isocratean critic of theoretical philosophy in the *Protrepicus* is arguing that sciences preoccupied with theoretical objects (such as the objects of music understood as a mathematical science) do not prepare the student for any purposeful activity, as compared to, for example, training in music that prepares one for the activity of playing music. In Aristotle’s framework, however, observation and contemplation of theoretical objects is itself an activity, one with an intrinsic end. For φιλομάθεια see also *NE* III 13.1117b29.

<81.5-24: commentary>

81.5-7 attribution and voice: a brief and abrupt bridge passage in which Iamblichus makes a rapid transition to the defense of theoretical philosophy; cf. *DCM* XXVII 88.25; this bridge passage differs from the corresponding one at *Protr.* VI 37.26-38.3 by announcing only that mathematical sciences are capable of being acquired, whereas the wider domain of theoretical science is under consideration in VI, including “sciences about nature and the rest of truth”.

81.5-24 attribution, and voice: A new speech and speaker here begins replying to the previous objections to theoretical mathematical sciences. The speech overlaps in part with *Protr.* VI 38.3-39.4 (for details, see below, and our analysis of the *Protr.* VI = *DCM* XXVI overlap on pp. NNN).

Beginning as it does after the extremely abrupt transition at 81.5-7, and the effect of this speech is jarring. This is because the same person can hardly be both criticizing the theoretical sciences as useless and impractical, and at the same time defending them as beneficial and practical. We are forced to the conclusion that these excerpts come from opposing speeches, i.e. a dialogue in which there were at least two speakers. Jaeger, Düring and in general all those who have treated the work not a dialogue but instead a letter or treatise have failed to account adequately for the evidence contained in this chapter of Iamblichus. Once we assume that it was a dialogue, the abrupt transition from a detailed attack on theoretical philosophy into a detailed defense of mathematical philosophy makes perfect sense.

81.7-16 is a modified overlap of *Protr.* VI 38.3-14. For detailed commentary, see notes on *Protr.* VI. The modification introduced in the *DCM* is that a citation is replaced by

paraphrase, to focus on mathematics: *Protr.* 39.7-9 “And good things are determinate and organized more than bad things, just as a fair person is <more> than a foul person, for they necessarily have the same mutual difference.” → “And there are determinate and ordered things among the immovable mathematical forms.” *DCM* 81.11-12.

81.7 ἀμότερα: Not present in the overlapping passage at *Protr.* VI 38.3-4, but it supports the interpretation that Aristotle’s comparing two (not three) kinds of science there (see note *ad loc.*). It is unclear whether to retain the word in both versions (seeing its loss in *Protr.* VI as an accident), or to retain it in *DCM* 26 only (seeing this as a difference caused by Iamblichus), or to remove it from both versions (seeing it as a marginal gloss incorporated into the text).

81.11-12 is a bridge passage, corresponding to a passage that Iamblichus quoted at *Protrepticus* VI 38.7-10; as above at 81.5-7, in this bridge passage Iamblichus restricts the range of the argument that results in *DCM* XXVI to the mathematical entities, whereas in the other presentation in his *Protrepticus*, he refers to ‘good’ things being more determinate and organized. So do we see here Iamblichus in the act of falsifying his source text? No, he has referred to the very material of his own *Protrepticus*, we think, rather than re-excerpt afresh from a copy of Aristotle’s work, and helps himself to the narrower idea that mathematical objects are among the good things that are determinate and organized, an idea with which Aristotle argued for in the *Protrepticus*. Iamblichus’ paraphrase says a bit less than the original must have, in order to focus on an aspect of what the source text was saying; it is a narrower focus, not a distortion.

81.16: At *Protr.* VI 38.14 there is a further phrase: στοιχεῖα δὲ τῶν ὀνομαζομένων συλλαβῶν. This phrase has proved extremely difficult to construe, and these two factors motivated its deletion by Kiessling, followed by Pistelli; see note *ad loc.*

81.16-20: a summarizing bridge passage in a place where there was a citation at *Protrepticus* VI 38.14-39.8, as also happened above at 81.11-12. In this case, both passages use as a central move in an *a fortiori* argument that one item is ἀρχικώτερα, and therefore better. In *DCM* 26 it is mathematical entities that have this status (because they are more simple than other things), whereas in *Protrepticus* VI, the argument is that soul is better than body because it is ἀρχικώτερα. Again, as at 81.12-16, the question arises whether we are seeing Iamblichus in the acting of modifying his source? We think the more likely explanation is that his source text contains both phases of argument, first the one about the soul, and then the one about mathematical entities, which is apparently being set up by the remark at VI 38.20-22 that since we are capable of acquiring the discipline of the soul, “since surely we are also capable of acquiring knowledge of things of which our ignorance is greater and cognition is harder to come by,” referring to mathematical science and physical speculation. But this question, whether Iamblichus helps himself surreptitiously to the possibility of slightly modifying his source text to make it more relevant to his theme in *DCM*, is not such as to admit of a definitive answer. If he did engage in surreptitious changes, that would generally undercut his reliability as a source for reliable paraphrase as well as for citations.

81.20-24: is literally identical to *Protr.* VI 38.22-39.4.

81.23 [καὶ] τᾶλλα In the other version of this material (*Protr.* 39.3) the conjunction is missing, and the text is easier to construe without it.

<81.24-82.13: commentary>

81.24-82.1 attribution and voice: A typical Iamblichean transition, corresponding to a gap in the excerpt quoted at *Protr.* VI at 39.4-39.8. One can closely compare the compression and reformulation here relative to 39.813. Here in the *DCM* Iamblichus substitutes "knowledge of mathematics" for "knowledge of the truth and of the virtue of the soul" in *Protr.* VI-- but what Aristotle goes on to say in both cases (even the *DCM*) talks about philosophy in general, not about knowledge of mathematics specifically. Iamblichus has also here substituted the vacuous remark that "reason and wisdom lead among goods" for the different point made in *Protr.* VI about knowledge of the truth and the soul being the most beneficial kinds of knowledge.

82.1: ἡγεῖται τῶν ἀγαθῶν: A similar thought and expression is found in Plato, *Leg.* 730c1: ἀλέθεια πάντων ἀγαθῶν ἡγεῖται.

81.24: [καὶ] In the parallel version of this passage, at *Protr.* 39.11, we find the reading preferable without the extraneous καὶ, which seems to have entered by scribal corruption.

82.1-11 attribution and voice: This passage is modified version of the excerpt at *Protr.* VI 39.16-40.1. For detailed commentary, see the notes *ad loc.* The modifications are as follows. The opening sentence is modified to remove the rhetorical question. *Protr.* 39.17-18 "What standard do we have, what criterion of good things, that is more precise than the intelligent man?" → "There is no other criterion or standard of good things more precise than the intelligent man." *DCM* 81.1-3. Later, a short clause with unnecessary precision omitted. *Protr.* 39.18-20 "For all that this man will choose, if the choice is based on his knowledge, are good things and their contraries are bad." → "For all that this man will choose [...] are good things and their contraries are bad." *DCM* 82.3-4.

This is highly significant as the clearest case where we see Iamblichus comb out of his *DCM* traces of dialogue that had been left to stand in his *Protrepticus* citations, although he must have carefully removed most of those (as in the Plato passages of *Protr.* 13-18), if we are correct in seeing Aristotle's original *Protrepticus* as a dialogue.

82.2 ἀκριβέστερος: Here the *Protr.* version (39.17) seems to be better than that of *DCM*. The argument makes perfect sense that one should do theoretical philosophy because is no more precise standard of good. Superlatives are used, however, in what is apparently Iamblichus' bridge passage immediately preceding at (μέγιστόν ... ὠφελιμώτατον, 81.24-25). This may indicate that the original argument argued not only that that theoretical and mathematical philosophy is more precise than the other sciences, but, further, that mathematical philosophy is the most precise and beneficial of all the sciences.

82.5 <τὰ> κατὰ: that the version of the text at *Protr.* VI 39.21 has the definite article, which should be supplied here as well.

82.11-13 attribution: Must be an Iamblichean transition, because there is a gap in the excerpt here relative to *Protr.* VI at 40.1-11.

<82.14-83.2: commentary>

82.14-83.2 attribution and voice: 82.14-17 reviews the conclusion of the previous argument that theoretical philosophy has proven beneficial; and announces the following argument, that it is easy. These are discussed programmatically in the dialogue: see notes on *Protr.* VI 37.22-26; see also below at 83.2-5.

The passage contains three first-person references that seem to be in the voice of a speaker in the dialogue: “I think” (νομίζω, 82.15), “I was convinced” (πέπεισμα, 82.17), and “it seems to me” (μοι δοκεῖ, 82.21). In the overlapping passage in *Protr.* VI the second of these has been flattened to an impersonal verb (πεισθείη, 40.15) but the other two first-person references are retained (40.13, 40.19). (Iamblichus seems to have left two fossils of dialogue in the excerpt in *Protrepticus* VI, but three here in *DCM* XXVI. This is thus an important case for seeing how Iamblichus does and does not modify his excerpts.

82.17-83.2 is literally identical (with the exception below) to *Protr.* VI.40.15-41.2, which we consider to be a speech authored by Aristotle and put into his own voice (see commentary *ad loc.*). Proclus, in *Euc.* I 1, 9.28.13-22 the arguments to Aristotle with the words “as Aristotle somewhere says” (28.14-15).

82.18 τοῖς φιλοσόφοις: φιλοσόφουσι *Protr.* VI 40.16.

82.19 <εἶς> τὰς: Festa follows the text of the parallel version here (*Protr.* 40.17), correctly.

<83.2-22: commentary>

83.2-5 attribution and voice: 83.2-5 is a navigational passage, making a transition away from the arguments to the effect that philosophy is “possible” into the next programmatic topic, that philosophy is beneficial (“the greatest of goods”), and finally, easy to acquire. See the above navigational passage at 82.14-17 and the note describing this program of topics at *Protr.* VI 37.22-26.

83.5 φρόνησις] φιλοσοφία *Protr.* VI 41.4. The two different intellectual qualities that are named in the same place in the same sentence in the two different versions suggests that in one or both cases Iamblichus is paraphrasing, not citing.

83.6-22 attribution and voice: The argument is an anthropological account of the development of the arts that adds details and depth to Aristotle’s argument at *Protrepticus* VI 40.15-20 = *DCM* XXVI 82.17-22 and *Metaph.* I 1.982b11-28 + I 2.982b11-28.

83.7 ἡ περὶ τὴν ἀλήθειαν ἀκριβολογία: See above μάλιστα ἠκριβωκέναι at 79.9-10 and below κάλλει καὶ ἀκριβεία at 83.24-25; and the note on ἀκριβέστερος at *Protr.* VI 39.17 (and cf. VI 40.19 and *DCM* XXVI 79.10, 83.7, 83.24-25, *DCM* XXVII 86.8). On the NN of mathematics, see *Metaphysics* II(α) 995a15 and generally a6-20.

83.7-8 μετὰ γὰρ τὴν φθορὰν καὶ τὸν κατακλυσμὸν: See note on *Protr.* VI 40.17-19 about Aristotle’s theory of cultural development in relation to his meteorological-cosmological theories about cyclical cataclysms. Aristotle believed that the earth was subjected to periodic catastrophic flooding, as he explains at *Meteor.* I 14.352a33 (where he reflects directly to the flood of Deucalion); see also *Phys.* IV 13.222a23, and [Ar.] *Prob.* XV 16.910a35. He also several times mentions the rediscovery of knowledge and wisdom lost after such cataclysms at *Cael.* 270b19, *Meteor.* 339b29, *Metaph.* 1074b10, and *Pol.* 1264a3, and 1329b25.

Philoponus (in *Nic. Arith. Intro.* I.1.5-49) recounts a story about the rise of human arts and wisdom after destruction and cataclysm, in a passage in which he references Aristotle and specifically refers to the flood of Deucalion (see text, translation, and commentary at pp. NNN).

For the flood of Deucalion see also Apollodorus, *Library*, 46-48; Hyginus, *Fables*, 153; and Ovid, *Met.* 253-312.

83.9 φιλοσοφείν] φροντίζειν: The account of the development of theoretical science through leisure would be undermined if the humans forced to focus on food and survival are already said to do philosophy. It seems that the received text has been corrupted away from a verb like φροντίζειν (which is suggested by the parallel in Proclus in *Euc.* I, prol. I, chapter 9: “it is generally when they stopped being concerned (φροντίδος) about the necessities that humans turned towards mathematics” (29.1-2). This verb means “to be thoughtful” in the sense of being “concerned” or “anxious”, and so gives just the right sense: they “were compelled to be concerned about their food and staying alive”. Cf. Isoc., *Evag.* 41).

83.11 μουσικήν καὶ τὰς τοιαύτας <sc. τέχνας>: See above at 80.13: μουσικήν καὶ τὰς ἄλλας ἐπιστήμας. It is interesting that music is not explicitly mentioned in the parallel account in *Metaphysics* I.

83.12 ἐπεχείρησαν φιλοσοφείν: See note on ἐπιχειροῦσιν above at 79.5.

83.13-21: Regarding the general idea of progress in the arts and sciences, and the especially rapid recent progress in mathematical and theoretical philosophy, see above 82.17-22 = *Protr.* VI 40.15-20 (and see notes *ad loc.*). The present passage is an important piece of fourth century evidence for the rapid advance in mathematical sciences in the Old Academy, cited by Burkert for this purpose (*Lore and Science*, 422-423), who notes of the passage that its “Aristotelian origin is guaranteed by Cicero. *Tusc.* 3.69”. “Aristotle in upbraiding the philosophers of old for thinking, according to him, that thanks to their genius philosophy had reached perfection, says that they had been guilty of extreme folly or boastfulness; all the same he adds that he saw that, as a consequence of the great advance made in a few years, philosophy would be absolutely complete” (tr. King, LOEB). But one need not turn to Cicero for the same thought: in the *Politics* Aristotle expresses a progressive view of the state of science in general terms: “we must not fail to keep in mind the length of time and multitude of years in which these things, if they had been good would certainly not remain unknown; for almost everything has been found out, though in some cases what is known has not been systematized, and in other cases man do not make use of the knowledge which they have” (II 1264a1-5; cf. II 1268b36-1269a8; Lovejoy and Boas, *Primitivism*, 178-179).

83.21-22 τὸ γὰρ τῇ γενέσει ὕστερον, οὐσίᾳ καὶ τελειότητι προηγείται: See *Protr.* IX 51.16-23 for the principle that the end has priority in substance over what is earlier in the order of generation. In that passage, Aristotle is referring to the natural development of an organism, and this principle is often invoked in that context (e.g., *PA* I 1 640a19-26, 641b23-642a1, II 1 646a25-27, *GA* II 1 734a16-32, ii 6 *passim*; see also Johnson, *Teleology*, 165-171). It is striking to see the same biological principle applied here to cultural history. This suggests a reason for the late development of theoretical sciences different than the mechanism described in the *Politics* for the evolution of the practical arts (according to which better ways of doing things will inevitably have been discovered, especially in contexts where it is pragmatically necessary to do those things).

<83.23-84.7: commentary>

83.23-84.7 attribution and voice: The speech contained in this section is directed at the conclusion that theoretical sciences are valuable in and of themselves. But the style of argument,

repetition and lack of steady argumentative progression, and the excessive focus on mathematics suggests that Iamblichus may be adapting his source material here to better fit with his greater aim in the *DCM* to defend theoretical mathematics in particular. 83.23-25 looks much like a navigational bridge passage, referring to what has been shown, and offering another argument in support. The difficulty here is that the argument in question is supposed to be about the precision and beauty of mathematics (83.24-25), but no argument to that effect has been made here in

83.24-25 κάλλει καὶ ἀκριβείᾳ: An extremely suggestive phrase which seems to us to be deliberately worded as an exact counterpart to the practical excellence of καλοκἀγαθία, which was important to Isocrates, and mentioned in a rebuttal to him excerpted at *Protr.* IX 53.1 (see note *ad loc.*). For ἀκριβείᾳ see note and cross-references above at 79.9-10. As for κάλλει: so far, there has been no argument about the “beauty” of mathematics in Iamblichus. In a passage excerpted by Iamblichus at *Protrepticus* VI 38.5, “order and definiteness” are mentioned, and these are two of the three forms of beauty according to *Metaphysics* XIII 3 (the third being symmetry). But no argument about the existence of beauty in mathematics is spelled out there by Iamblichus, who focuses instead on the defense of theoretical philosophy against the charge of uselessness; see note *ad loc.* Proclus, however, in his *Commentary on the First Book of Euclid’s Element* (Prologue I, chapter 9.26.10-27.10—see notes *ad loc.*) paraphrases and connects not only Aristotle’s arguments in response to the charge of uselessness, but also an argument based on the presence of the beautiful among mathematical objects (since they exhibit order, symmetry and definiteness in an exemplary way). He seems to use Aristotle’s *Protrepticus* as a source for his paraphrase since he says they are arguments “by which Aristotle attempts to persuade us”, which would be an odd way to refer to the mathematical books of the *Metaphysics*, but a good way to refer to the *Protrepticus*, a work in which Aristotle tried to persuade the youth to do theoretical philosophy.

84.1-7: The passage offers another argument to the same conclusion, that theoretical sciences are intrinsically valuable. The hierophantic tone of the phrase “liberated from our bodily nature” suggests the possibility of Iamblichean authorship, as does the fact that the argument here is compressed and sketchy, and probably contains terminology not paralleled in the Aristotle Corpus (see below).

84.3-4 σωματοειδοῦς: This term is used by Plato (*Phd.* 81b5, c4, e1, 83d5, 86a2; *Tim.* 31b4, 36d9; *Pol.* 273b4; *Rep.* 532c7), but does not appear in the Aristotle Corpus, except in [Ar.] *Prob.* 936b35. It is later common in the doxographic tradition, including some fragments of Theophrastus, and in Plutarch, Aetius, etc.

<84.7-20: commentary>

84.7-10 attribution and voice: 84.7-10 seems to summarize the conclusion of all the preceding arguments presented in *DCM* XXVI, that mathematical skills are useful for our way of life, and the issue of usefulness is picked up in the next part of *Protrepticus* VII. This sentence at least may contain an indication of the contents of the source text.

84.11-20 attribution and voice: 84.11-14 asserts, abruptly and surprisingly, that the mathematical skills just celebrated for their usefulness “are worth little effort” and then we are treated to neo-pythagorean (neoplatonic) considerations expressed in terminology alien to the Aristotle corpus. 84.14-20 then seems to be a typical kind of Iamblichean closing, summarizing the conclusions of the chapter and reminding us of its purpose in accordance with the title.

84.12 ἡ κάθαρσις ἐστὶ τῆς ἀθανάτου ψυχῆς: The use of κάθαρσις here does not correspond to Aristotle's usage, notwithstanding the wide semantic range that term might have in the *Poetics* and *Politics*. The idea that early Pythagoreanism can be characterized by preoccupation with "purification of the soul" (Döring, *AGP* 1892, 505; Cornford, 'Mysticism and Science', Burnet, *Early Greek Philosophy*, 97) has been rejected by more recent scholars as a construction of later antiquity (Burkert, *Lore and Science*, 211-213; Zhmud, *Early Pythagoreans*, 16).

84.13 ἡ τοῦ νοῦ περιγωγὴ πρὸς τὸ νοητόν: περιγωγὴ: This term is used by Plato (*Pol.* 270a; *Rep.* 518d4, e4, 521c6, 533d3), but does not appear in the Aristotle Corpus, except in [Ar.] *Mund.* 391b18, 399a2. It is common in Plutarch and Galen.

84.13-14 καὶ ἡ μετουσία τῆς τοῦ ὄντος ἐνεργείας: The term μετουσία, though extremely common after Plutarch, does not occur in the Aristotle Corpus. But it occurs twice in a fragment attributed to Aristotle's lost work *On Ideas* as reported by Alexander of Aphrodisias, in *Metaph.* 83.34-89.7 (frag. 188, Rose 1886; frag. 4, Ross, OCT) at 83.36 and 85.5.

84.16-17 πρὸς τὸ τέλος τῆς εὐδαιμονίας οὐκ οἶδ' εἴ τις ἄλλη μέθοδος οὕτω συναίρεται: Some of the terminology here was probably used in the source text. It is not clear if Iamblichus is speaking in his own voice with the first person verb, or if this could be an un-expunged vestige of dialogue.

84.18-19 ψευδεῖς οἱ ἐναντίοι λόγοι πεφύνασιν: This was the point of the chapter as described by Iamblichus in its title.

84.19 τὰ μαθήματα: Cf. *DCM* XXV 78.22: τὰ μαθήματα τῶν πραγμάτων.